A Derivation of mixed advertising strategy oligopoly equilibrium

Here, we use Expressions (13) and (14) to derive the mixed advertising strategy oligopoly equilibrium discussed in Section 4.

Setting $\pi^a(v) = \pi^a(p)$ for $p \in [p^*, v]$, we can use Expression (13) to solve for $F^a(p)$ in terms of $\alpha$.

$$F^a(p) = 1 - \left( \frac{v}{p} - 1 \right)$$

$$\times \left[ (1 - \alpha)^{N-1}[\gamma(1)]^2 + \sum_{k=1}^{N-2} \binom{N-1}{k} \alpha^k(1 - \alpha)^{N-1-k}[\gamma(k+1)]^2 + \alpha^{N-1} \right]$$

(A1)

Setting $F^a(p^*) = 0$ yields $p^*$ in terms of $\alpha$.

$$p^* = \frac{(1 - \alpha)^{N-1}[\gamma(1)]^2 + \sum_{k=1}^{N-2} \binom{N-1}{k} \alpha^k(1 - \alpha)^{N-1-k}[\gamma(k+1)]^2 + \alpha^{N-1}}{(1 - \alpha)^{N-1}[\gamma(1)]^2 + \sum_{k=1}^{N-2} \binom{N-1}{k} \alpha^k(1 - \alpha)^{N-1-k}[\gamma(k+1)]^2 + \alpha^{N-1}(2N - 1)}$$

Similarly, setting $\pi^n(p^*) = \pi^n(p)$ for $p \in [p, p^*]$, we can use Expression (14) to solve for $F^n(p)$ in terms of $\alpha$ and $p^*$ as follows.

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\[ F^n(p) = 1 - \left( \frac{p^*}{p} - 1 \right) \times \left[ \frac{(1 - \alpha)^{N-1} + \sum_{k=1}^{N-1} \binom{N-1}{k} \alpha^k (1 - \alpha)^{N-1-k} [(1 - \mu)^2 + 2k(1 - \mu)\gamma(k)]}{2(N-1)(1 - \alpha)^{N-1} + \sum_{k=1}^{N-2} \binom{N-1}{k} \alpha^k (1 - \alpha)^{N-1-k}2(N - 1 - k)(1 - \mu)^2} \right] \]  

(A2)

Setting \( F^n(p) = 0 \) yields \( p \) in terms of \( p^* \) and \( \alpha \).

\[ p = \frac{(1 - \alpha)^{N-1} + \sum_{k=1}^{N-1} \binom{N-1}{k} \alpha^k (1 - \alpha)^{N-1-k} [(1 - \mu)^2 + 2k(1 - \mu)\gamma(k)]}{(1 - \alpha)^{N-1}(2N - 1) + \sum_{k=1}^{N-1} \binom{N-1}{k} \alpha^k (1 - \alpha)^{N-1-k} \{ (1 - \mu)^2 [2(N - k) - 1] + 2k(1 - \mu)\gamma(k) \}} \]

At \( p^* \), firms must be indifferent between advertising and not advertising. Setting \( \pi^n(p^*) = \pi^a(p^*) \), we can implicitly solve for \( \alpha \) in terms of \( A, \mu \) and \( N \).

\[ \frac{N^2 A}{p^*} = \alpha^{N-1} \left\{ 1 - (1 - \mu)^2 + 2(N - 1) [1 - (1 - \mu)\gamma(N - 1)] \right\} + (1 - \alpha)^{N-1} \left[ [\gamma(1)]^2 - 1 \right] + \sum_{k=1}^{N-2} \binom{N-1}{k} \alpha^k (1 - \alpha)^{N-1-k} \times \left\{ [\gamma(k + 1)]^2 - (1 - \mu)^2 + 2k \left[ [\gamma(k + 1)]^2 - \gamma(k)(1 - \mu) \right] \right\} \]

(A3)

Checking for deviations

We need to check for two types of deviations: (1) price deviations and (2) advertising deviations. We start by showing when firms have no incentives to deviate from the pricing strategy described above. There are two types of deviations that we must consider. First, when a firm advertises, it must not wish to deviate to a price below \( p^* \). Second, when a firm does not advertise, it must not wish to deviate to a price above \( p^* \).

When a firm advertises, it does not want to lower prices below \( p^* \): By deviating to a price \( p < p^* \), the firm expects deviation profits \( \pi^a_d(p) \):

\[ \text{Note that we use the fact that } 2[N - 1 - (N - 1)](1 - \mu)^2/N^2 = 0, \text{ which says that when all other firms are advertising it is impossible for consumers to observe two firms that do not advertise.} \]
\[ \pi_d^a(p) = \frac{p}{N^2} \left\{ (1 - \alpha)^{N-1} \left[ [\gamma(1)]^2 + 2(N - 1)(1 - \mu)\gamma(1) (1 - F^n(p)) \right] + \alpha^{N-1} (2N - 1) + \sum_{k=1}^{N-2} \binom{N - 1}{k} \alpha^k (1 - \alpha)^{N-1-k} \times \left[ [\gamma(k + 1)]^2 (1 + 2k) + 2(N - k - 1)(1 - \mu)\gamma(k + 1) (1 - F^n(p)) \right] \right\} - A \]

(A4)

There are no incentives to lower prices to some \( p' < p^* \) if expected deviation profits are lower than the expected equilibrium profit, that is, if \( \pi^n(p^*) > \pi_d^a(p') \). This is the case whenever the derivative of expected deviation profits with respect to \( p \) is positive, so that lowering prices decreases expected profits. Substituting \( F^n(p) \) from Expression (A2) into Expression (A4), yields the following expression:

\[ \pi_d^a(p) = \frac{p}{N^2} \left[ \Psi_1 + \Phi_1 \Gamma_1 \left( \frac{p^*}{p} - 1 \right) \right] \]

where

\[ \Psi_1 \equiv (1 - \alpha)^{N-1}[\gamma(1)]^2 + \alpha^{N-1}(2N - 1) \]

\[ + \sum_{k=1}^{N-2} \binom{N - 1}{k} \alpha^k (1 - \alpha)^{N-1-k} [\gamma(k + 1)]^2 (1 + 2k) \]

\[ \Phi_1 \equiv (1 - \alpha)^{N-1}2(N - 1)(1 - \mu)\gamma(1) \]

\[ + \sum_{k=1}^{N-2} \binom{N - 1}{k} \alpha^k (1 - \alpha)^{N-1-k} 2(N - k - 1)(1 - \mu)\gamma(k + 1) \]

\[ \Gamma_1 \equiv \frac{(1 - \alpha)^{N-1} + \sum_{k=1}^{N-1} \binom{N - 1}{k} \alpha^k (1 - \alpha)^{N-1-k} ((1 - \mu)^2 + 2k(1 - \mu)\gamma(k))}{(1 - \alpha)^{N-1}2(N - 1) + \sum_{k=1}^{N-2} \binom{N - 1}{k} \alpha^k (1 - \alpha)^{N-1-k} 2(N - k - 1)(1 - \mu)^2} \]

(A5)

Taking the derivative of Expression (A5) with respect to \( p \) yields the following expression, which is independent of \( p \).

\[ \frac{d\pi_d^a}{dp} = \frac{\Psi_1 - \Phi_1 \times \Gamma_1}{N^2} \]

(A6)

As long as Expression (A6) is positive, the firm does not wish to decrease its price below \( p^* \).

**When a firm does not advertise, it does not want to increase prices above \( p^* \):** By deviating to a price \( p > p^* \), the firm expects deviation profits \( \pi_d^a(p) \):
\[
\pi_n^d(p) = \frac{p}{N^2} \left\{ (1-\alpha)^{N-1} + \alpha^{N-1} \left[ (1-\mu)^2 + 2(N-1)(1-\mu)\gamma(N-1) (1 - F^a(p)) \right] \right. \\
+ \sum_{k=1}^{N-2} \left( \frac{N-1}{k} \right) \alpha^k (1-\alpha)^{N-1-k} \left[ (1-\mu)^2 + 2k(1-\mu)\gamma(k) (1 - F^a(p)) \right] \left\} \right.
\]

\[ (A7) \]

The firm does not have an incentive to increase prices to some \( p' > p^* \) when the derivative of expected deviation profits with respect to \( p \) is negative, such that raising prices decreases expected profits. Substituting \( F^a(p) \) from Expression (A1) into Expression (A7), yields the following expression:

\[
\pi_n^d(p) = \frac{p}{N^2} \left[ \Psi_0 + \Phi_0 \Gamma_0 \left( \frac{v}{p} - 1 \right) \right]
\]

where

\[
\Psi_0 = (1-\alpha)^{N-1} + \sum_{k=1}^{N-2} \left( \frac{N-1}{k} \right) \alpha^k (1-\alpha)^{N-1-k}(1-\mu)^2 + \alpha^{N-1}(1-\mu)^2
\]

\[
\Phi_0 = \sum_{k=1}^{N-2} \left( \frac{N-1}{k} \right) \alpha^k (1-\alpha)^{N-1-k}2k(1-\mu)\gamma(k) + \alpha^{N-1}2(N-1)(1-\mu)\gamma(N-1)
\]

\[ (A8) \]

\[
\Gamma_0 = \frac{(1-\alpha)^{N-1}[\gamma(1)]^2 + \sum_{k=1}^{N-2} \left( \frac{N-1}{k} \right) \alpha^k (1-\alpha)^{N-1-k}[\gamma(k+1)]^2 + \alpha^{N-1}}{\sum_{k=1}^{N-2} \left( \frac{N-1}{k} \right) \alpha^k (1-\alpha)^{N-1-k}2k[\gamma(k+1)]^2 + 2(N-1)\alpha^{N-1}}
\]

Taking the derivative of Expression (A8) with respect to \( p \) yields the following expression, which is independent of \( p \).

\[
\frac{d\pi_n^d}{dp} = \frac{\Psi_0 - \Phi_0 \ast \Gamma_0}{N^2}
\]

\[ (A9) \]

As long as Expression (A9) is negative, the firm does not wish to increase prices above \( p^* \).

**Checking for advertising deviations**

We next need to confirm that firms do not want to alter their advertising strategy. In particular, to place bounds on the set of mixed advertising strategy equilibria, we construct no-deviation conditions from pure advertising strategy equilibria (\( \alpha = 0 \) and \( \alpha = 1 \)). Following the procedure used to derive Proposition 1 we can show that when firms either advertise with certainty or not at all, the equilibrium price distribution equals \( F^b(p) = 1-(v/p-1)/(2N-2) \), which has lower bound \( p^b = v/(2N-1) \). Equilibrium expected profits equal \( \pi^b = v/N^2 \) when \( \alpha = 0 \) and \( \pi^b - A \) when \( \alpha = 1 \).

**Deviation from \( \alpha = 1 \):** Because equilibrium expected profits in this case equal to \( v/N^2 - A \), it must be that \( A < v/N^2 \). Moreover, by deviating to not advertising, a firm obtains deviation
When all of its rivals advertise, a firm that deviates to not advertising ceases to be prominent to all prominent rivals and ends up with a lower fraction of captives \((1 - \mu)^2/N^2\) and competes for \(2(N - 1)(1 - \mu)\gamma(N - 1)/N^2\) shoppers while avoiding the cost of advertising, \(A\). The firm does best by deviating to no advertising when it is charging \(p\), in which case it attracts all shoppers.\(^2\)

Firms do not wish to deviate if \(\pi(p)\) is greater than \(\pi^d(p)\). That is, if

\[
\frac{p}{N^2} (2N - 1) - A > \frac{p}{N^2} [(1 - \mu)^2 + 2(N - 1)(1 - \mu)\gamma(N - 1)]
\]

which reduces to,

\[
A < \frac{\mu[2(N - 1) + \mu]}{(2N - 1)N^2} v \equiv A_L
\]

**Deviation from \(\alpha = 0\):** By deviating to advertising, a firm becomes prominent and earns deviation profit

\[
\pi^d(p) = \frac{p}{N^2} [(\gamma(1))^2 + 2(N - 1)(1 - \mu)\gamma(1)(1 - F_b(p))] - A
\]

In this case, the firm does best by deviating to advertising when it is charging \(v\) and, therefore, only sells to its captives.\(^3\) Thus, the no-deviation condition is

\[
\frac{v}{N^2} > \frac{v}{N^2} [(\gamma(1))^2 - A]
\]

which can be rewritten as

\[
A > \frac{[(\gamma(1))^2 - 1]}{N^2} v \equiv A_H
\]

Therefore, whenever \(A \in (A_L, A_H)\), firms play mixed advertising strategies.

\(^2\)Subtracting pure strategy profit \(\pi^b\) from Expression (A10), the derivative of the gains from deviating with respect to \(p\) is:

\[
\frac{d(\pi_d(p) - \pi^b(p))}{dp} = (1 - \mu)^2 - 1 + [2(1 - \mu)(N - 1 + \mu) - 2(N - 1)][1 - F_b(p) - pF_b'(p)] = \frac{-\mu (1 - \mu)}{N(N - 1)} < 0
\]

where the second equality follows by substituting \(F_b(p)\).

\(^3\)Subtracting \(\pi^b\) from Expression (A13), the derivative of the gains from deviating with respect to \(p\) is:

\[
\frac{d(\pi^d(p) - \pi^b(p))}{dp} = [(\gamma(1))^2 - 1 + [2(N - 1)(1 - \mu)\gamma(1) - 2(N - 1)][1 - F_b(p) - pF_b'(p)] = \frac{\mu [1 + (N - 1)\mu]}{N} > 0
\]

where the second equality follows by substituting \(F_b(p)\).
B Sequential duopoly

In the model of Section 2, we supposed that firms simultaneously set their advertising and pricing strategies. However, if we view advertising decisions as ones that require more long term planning than pricing, it may be more appropriate to suppose that firms first simultaneously decide whether or not to advertise, then after observing the advertisement decisions they choose prices, following which consumers form consideration sets and make purchasing decisions. As we discuss in this section, the main results from Section 2 persist, though market segmentation is no longer absolute: that is, instead of exclusively focusing on different consumer segments, when one firm advertises and the other does not, the advertising firm focuses more on its captives and less on shoppers, whereas the other focuses on its diminished number of captives and more on shoppers than its advertising rival.

More precisely, in this sequential advertising and pricing game, there are three alternative advertising outcomes in the first stage: both firms advertise, neither does so, or one firm advertises whereas the other does not. In subgames where both firms advertise or neither does so, the equilibrium outcome is the same as in the pure strategy cases in Theorem 1. In contrast, suppose that firm $i$ advertises, whereas firm $j$ does not. In this case, if we suppose that price dispersion continues to persist, with the advertising and non-advertising firms drawing prices from respective continuous distribution functions $F_a(p)$ and $F_n(p)$ (not necessarily the ones derived in Section 3), then the profit functions of the advertising and non-advertising firms may be written as, respectively,

$$\pi^a(p) = \frac{p}{4} \left[ (1 + \mu)^2 + 2(1 - \mu^2) (1 - F_n(p)) \right] - A,$$  \hspace{1cm} (B16)

$$\pi^n(p) = \frac{p}{4} \left[ (1 - \mu)^2 + 2(1 - \mu^2) (1 - F^a(p)) \right].$$  \hspace{1cm} (B17)

Equations (B16) and (B17) rule out the equilibrium studied in Section 3, in which an advertising firm always sets a higher price than its non-advertising rival. Suppose to the contrary that $F^a(p)$ and $F^a(p)$ had respective supports $[p^a, p^n]$ and $[p^n, v]$. Then Equations (B16) and (B17) reduce to, respectively, $\pi^a(p) = (1 + \mu)^2 p/4 - A$ and $\pi^n(p) = [(1 - \mu)^2 + 2(1 - \mu^2)]p/4$. But in this case, both firms would wish to deviate to $v$, a contradiction.

**Lemma B1.** *In the equilibrium of the subgame of the sequential advertising and pricing game in which one firm advertises and the other does not, the supports of firm pricing distributions are the same, do not have any breaks, and are bounded from above by $v$. In equilibrium, one firm has an atom at $v$.*

For concision, we state Lemma B1 without proof, referring the reader, instead, to Narasimhan S6.
(1988) and Astorne-Figari and Yankelevich (2014), which derive similar results in asymmetric extensions of Varian’s (1980) model of sales. With these restrictions on firm pricing, it becomes straightforward to derive firms’ equilibrium price distributions and profits in the subgame in which only one firm advertises by following a similar approach to that used in Sections 2.3.1 and 3.

**Proposition B1.** In the subgame perfect Nash equilibrium outcome of the game of advertising, then pricing, when \( A \in (\mu(3+\mu) - 2] \mu v/[4(3-\mu)], \mu(2+\mu)v/4 \), only one firm advertises in equilibrium. Firms price according to

\[
F^n(p) = \left[ 1 + \frac{1+\mu}{2(1-\mu)} \right] \left( 1 - \frac{p}{v} \right), \quad F^a(p) = \left[ 1 + \frac{1-\mu}{2(1+\mu)} \right] \left( 1 - \frac{p}{v} \right)
\]

over support \([p, v] \), where \( p = (1+\mu)v/(3-\mu) \). In equilibrium, the advertising firm has an atom at \( v \) and \( F^b(p) \geq F^n(p) > F^a(p) \). When only one firm advertises, it always earns higher profit than in the baseline, whereas its rival earns higher profit for \( \mu \in (0, 0.56) \).

**Proof.** Working backward, suppose that only one firm advertises in equilibrium. From Lemma B1, we know that it must be that \( \pi^n(p) = \pi^a(p) \) for all \( p \in [p, v] \). Using Expression (B17), we obtain \( F^a(p) \) in the statement of Proposition B1. Likewise, \( \pi^a(p) = \pi^a(p) \) for all \( p \in [p, v] \), so that using Expression (B17), we obtain \( F^n(p) \) in the statement of Proposition B1.

Note that both \( F^n(p) \) and \( F^a(p) \) equal to zero at \( p \). Moreover, setting \( F^n(p) = 1 \) and solving for \( p \) yields \( p = (1+\mu)v/(3-\mu) \). Substituting this expression into \( F^a(v) \) yields \( F^a(v) = (3+\mu)(1-\mu)/[(3-\mu)(1+\mu)] \in (0, 1) \), which implies that the advertising firm has an atom at \( v \).

Comparing \( F^n(p) \) and \( F^a(p) \) yields \( F^n(p) \geq F^a(p) \) if and only if \( (1+\mu)^2 > (1-\mu)^2 \), which holds for all \( \mu \in (0, 1) \). Further, substituting the expression for \( p \) into \( F^n(p) \) and comparing to Equation 1 yields \( F^b(p) > F^n(p) \), where \( F^b(p) \) is the price distribution following the subgames in which both firms advertise in the first stage or neither firm advertises in the first stage.

To see that it is a subgame perfect outcome for one firm to advertise whenever \( A \in (\mu(3+\mu) - 2] \mu v/[4(3-\mu)], \mu(2+\mu)v/4 \), we must show that in this outcome, the advertising firm earns more profit than when neither firm advertises and that the non-advertising

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4Roughly, firms make no profit above \( v \) and, with its advertising action fixed, a firm pricing below its rival’s lower bound gets the same number of captives and all shoppers at any such price and stands to profit by shifting its price upward. Firms cannot have an atom at the same price by a similar atom undercutting argument used in Lemma 1. Firms cannot have breaks in their supports because their rivals expect to capture the same number of shoppers everywhere along the break, implying that profits are increasing in price along the break, a contradiction of mixed strategy equilibrium. Finally, convex supports imply that firms cannot have an atom anywhere below \( v \) because otherwise a firm can profitably shift mass above the atom slightly below it to capture shoppers with strictly higher probability.
firm earns more profit than when both firms advertise. This requires that \( \pi^a > v/4 \) and \( \pi^n > v/4 - A \) (expected profits in the pure advertising strategy equilibrium with \( \alpha = 0, v/4 \), and the pure advertising strategy equilibrium with \( \alpha = 1, v/4 - A \), from Theorem 1. The respective comparisons lead to the specified range for \( A \), which is illustrated in Figure B1 for \( \mu \in (0, 1) \). From this, it immediately follows that in this range, the advertising firm always earns higher profit than if its rival advertised or if neither firm advertised. It also follows that in this range, the non-advertising firm earns higher profit than if both firms advertised, but not necessarily if neither firm advertised.

Setting \( \pi^n > v/4 \), it follows that \( \pi^n \) is greater whenever \( \mu \in (0, 0.56) \). That is, for sufficiently low \( \mu \), both firms earn higher profits than in the baseline.

Proposition B1 implies that unlike in the simultaneous pricing and advertising game, when one firm advertises and the other does not in equilibrium, both price over the same support. However, although the market is not perfectly segmented as in Proposition 2, because \( F^a \) dominates \( F^n \) in the first order stochastic sense, as in Section 3, the advertising firm focuses more on catering to its larger number of captives, whereas its non-advertising rival prices more aggressively to increase its probability of capturing all shoppers.

![Figure B1: Equilibrium over parameter space \( \mu \times A \)](image)

Figure B1 captures the parameter space over which only one firm advertises in the sub-game perfect outcome. As in Section 2, this happens for an intermediate range of \( A \). When \( A > \mu(2 + \mu)v/4 \), neither firm advertises, and when \( A < [\mu(3 + \mu) - 2] \mu v/[4(3 - \mu)] \), both
do so in equilibrium. In each case, firms price according to the baseline distribution $F^b$ from Section 2.3.1 and both firms expect to set lower prices than when only one firm advertises. Intuitively, this occurs for the same reasons that firms set higher prices in Section 3: advertising diminishes the fraction of shoppers and allows firms to focus on different market segments.

In the light-gray region in Figure B1, the advertising firm always earns higher profits than it would without advertising, while the non-advertising firm earns higher profits than it would if both firms advertised (otherwise they would deviate in the first stage). Moreover, if $\mu$ is not too high (below approximately 0.56), the non-advertising firm earns higher profit than if neither firm advertised, meaning that as in Section 3, advertising for attention has the potential to raise aggregate and individual firm profits.

C Asymmetric advertising costs

Returning to the simultaneous pricing and advertising game in Section 2, without loss of generality, let us instead suppose that firm 1 faces advertising cost $A_1 = \sigma A$ for $\sigma \in [0, 1]$ and firm 2 faces advertising cost $A_2 = A$. Rather than undertaking an exhaustive analysis as we did in Section 2, we are interested in whether and when equilibria similar to those characterized in Sections 3 or B persist.

Assume first that $\sigma < 1$, so that $A_1 < A_2$. Suppose again, that $F^n_i(p)$ and $F^a_i(p)$ have respective supports $[p^n_i, p^a_i]$ and $[p^a_i, v]$, where subscript $i$ again denotes firm $i$. From Expressions (4) and (6), we know that $F^n_i(p)$ and $F^a_i(p)$ are functions of $\alpha_i$, which is endogenously determined by the cost of advertising. Using the procedure in Section 3, $F^n_i(p)$ and $F^a_i(p)$, it then follows that the price distribution of one firm contains an atom at $p^a_i$ (at the top of conditional distribution $F^n_i(p)$) and the price distribution of the other at $v$.\(^5\) However, if one firm has an atom at $p^a_i$, an advertising rival can profit by undercutting the rival, suggesting that complete market segmentation does not occur in equilibrium when advertising costs differ.\(^6\)

Consider, instead, a variant of the equilibrium outcome of Proposition B1, in which firm 1 advertises, whereas firm 2 does not.\(^7\) Although markets are not perfectly segmented in this equilibrium, it nevertheless represents a potentially profitable outcome to a counterfactual

\(^5\)Specifically, under our assumptions on $A_1$ and $A_2$ it can be shown that $F^n_2(p)$ contains the atom at $p^a_2$ and $F^a_1(p)$ contains the atom at $v$. Computations are available upon request from the authors.

\(^6\)Moreover, an alternative market segmentation equilibrium in which there is a gap between the upper-bound of the supports of $F^n_i(p)$ and the lower-bound of the supports of $F^a_i(p)$ cannot exist because the firm with the atom at $F^n_i(p)$ can profitably shift it slightly upward without losing customers.

\(^7\)Note that unlike in this section, that outcome was possible from firms playing symmetric strategies.
without advertising. Because the distribution functions in Proposition B1 are not functions of $A$, it follows that were firm 1 to advertise in equilibrium—while its rival does not—the distribution functions would be precisely those from Proposition B1. However, because we are now considering a simultaneous pricing and advertising game, we must adjust the conditions for a firm to deviate. As opposed to Section B—where a firm deviating from the asymmetric advertising outcome brings about the symmetric price distribution from Expression (1)—when firms make advertising and pricing decisions simultaneously, the advertising decision of a deviating firm is not observed by its rival before prices are determined.

**Proposition C2.** Suppose that $(2 - \mu)(1 + \mu)/\sigma > \mu(6 + \mu + 4\mu^2 + \mu^3)/(1 + \mu)$. Then, there exists an asymmetric equilibrium in which firm 1 advertises, firm 2 does not, and firms price according to

$$F_a^2(p) = \left[1 + \frac{1 + \mu}{2(1 - \mu)}\right]\left(1 - \frac{p}{v}\right), \quad F_a^1(p) = \left[1 + \frac{1 - \mu}{2(1 + \mu)}\right]\left(1 - \frac{p}{v}\right)$$

over support $[p, v]$, where $p = (1 + \mu)v/(3 - \mu)$.

**Proof.** The derivations of $F_a^1(p)$ and $F_a^2(p)$ proceed exactly following the same steps used to derive, respectively, $F_a(p)$ and $F_a(p)$ in the proof of Proposition B1. Instead, here, we focus on firms’ no-deviation conditions.

By deviating to not advertising, instead of earning $(1 + \mu)^2v/4 - \sigma A$, firm 1 earns $p[1/4 + (1/2)(1 - F_a^2(p))]$, which is decreasing in $p$ starting at $p$. Therefore, the no-deviation condition is

$$\frac{(2 - \mu)(1 + \mu)v}{4\sigma(3 - \mu)} > A \quad (C18)$$

Similarly, by deviating to advertising, instead of earning $(1 - \mu^2)(3 + \mu)v/[4(3 - \mu)]$, firm 2 earns $[3 + (10 - \mu)\mu]v/[4(3 - \mu)(\mu + 1)]$, which is increasing in $p$ up to $v$. Note that in computing firm 2’s deviation profit, we assume that it slightly undercuts firm 1’s atom at $v$. The no-deviation condition is

$$A > \frac{\mu(6 + \mu + 4\mu^2 + \mu^3)v}{4(3 - \mu)(1 + \mu)} \quad (C19)$$

Therefore, unless $(2 - \mu)(1 + \mu)/\sigma > \mu(6 + \mu + 4\mu^2 + \mu^3)/(1 + \mu)$ either firm 1 or firm 2 will wish to deviate.

Notice that this is almost precisely the equilibrium advertising and pricing outcome from Proposition B1. Although firms do not perfectly segment the market, firm 1 nevertheless caters primarily to captives, whereas firm 2 caters to shoppers by pricing more aggressively. What is notable about this equilibrium is that firm 1 is not passing down savings from a lower cost of advertising, instead taking advantage of its increased hold on consumer attention by

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pricing less aggressively than firm 2 or than either firm would in an equilibrium where neither firm advertises.

However, because \( \frac{\mu(6 + \mu + 4\mu^2 + \mu^3)}{(1 + \mu)} > (2 - \mu)(1 + \mu) \mu \) for all \( \mu \in (0, 1) \), the equilibrium in Proposition C2 does not exist unless \( \sigma \) is sufficiently low. This also implies that an equilibrium in which only firm 2 advertises does not exist, and moreover, that this asymmetric equilibrium does not exist when firms face the same advertising cost.

References

