Supplementary material: Tracks impacted field area simulation using kinematics and geometry for different equipment and operation scenarios

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Abstract
This supplementary material illustrates the mathematical derivation of the turning paths of harvester and baler, along with the ‘Up and Right’ and ‘Down and Left’ ABP turning cases in detail as examples. The geometrical path diagrams, the consolidated derived expressions of various critical parameters, the test conditions used in determining the turning cases of all the 17 turning cases of ABP, simulation results of the bale aggregation equipment track impacted area obtained for different field areas and bales/trip, as well as the ABP impacted area model prediction results are also presented.

A.1. Harvester and baler turning path derivation

Turn path derivation of harvester and baler on the headland are similar (Fig.A.1), but varies only on the equipment turning radius. The harvester turning radius is greater (6 m) than the baler (4.3 m). The harvester uses an “Ω-turn” as its turning radius is greater than its swath, while the uses an “U-turn” as its turning radius is smaller than the windrow spacing. Therefore, the harvester had to trace increased length of curvilinear track along the field edges than the baler.

Known parameters: Turning radius of the harvester and baler, swath width (s), number of swaths (n_s), and the field dimensions (Length = L and Width = W).

To find: Points A, I, F, D, H, G, and sweep angles.

Finding points A, E, F, and C.

\[ x_A = 0; \quad y_A = L \] (A.1.1)
\[ x_E = \frac{s}{2}; \quad y_E = 0 \] (A.1.2)
\[ x_F = \frac{s}{2}; \quad y_F = L \] (A.1.3)
\[ x_C = s; \quad y_C = L \] (A.1.4)

Finding point D:
With respective to point F,
Fig. A.1. Harvester and baler turning construction cases (r$_d$) — same as Figure 3 and is reproduced for ready reference.

\[ x_d = r_H; \quad y_d = 0 \]

With respective to the origin points,

\[ x_D = x_F - x_d; \quad y_D = y_F + y_d \]  \tag{A.1.5}

Finding the turning angle $\theta_t$:

In $\triangle$ DHI,

\[ DI = \frac{(x_d + 2(r_H))}{4}; \quad HI = \sqrt{DH^2 - DI^2} \]

Therefore,

\[ \theta_t = \tan^{-1}\frac{HI}{DI} \]  \tag{A.1.6}

Finding point H:

In $\triangle$ DHI,

\[ DH = r_H \]

\[ x_h = r_H \times \cos \theta_t; \quad y_h = r_H \times \sin \theta_t \]

\[ x_H = x_D + x_h; \quad y_H = y_D + y_h \]  \tag{A.1.7}

Finding point G:

In $\triangle$ DGC,

\[ DG = 2 \times r_H; \quad DC = r_H + \frac{s}{2}; \quad GC = \sqrt{DG^2 - DC^2} \]

With respective to the point F,

\[ x_g = 2 \times r_H \times \cos \theta_t; \quad y_g = 2 \times r_H \times \sin \theta_t \]

With respective to the origin,

\[ x_G = x_D - x_g; \quad y_G = y_D + y_g \]  \tag{A.1.8}

Finding sweep angle, $\gamma_s$:

In $\triangle$ GCD,

\[ \theta_g = 90 - \theta_t \]

Therefore,
\[
\gamma_s = 360 - (2 \times \theta_g)
\]  \hspace{1cm} (A.1.9)

Calculating the length of the whole curve, FF':

Length of curve of FH, HJ, and FJ:

\[
FH = \frac{\theta_t}{360} \times 2 \times \pi r_H; \ HJ = \frac{\gamma_s/2}{360} \times 2 \times \pi r_H \text{ (using the length of arc formula)}
\]

Length of curve FJ = FH + HJ

\[
FF' = 2 \times FJ
\]  \hspace{1cm} (A.1.10)

Calculating the linear travel distance, EF:

\[
EF = \sqrt{(x_E - x_F)^2 + (y_E - y_F)^2}
\]  \hspace{1cm} (A.1.11)

Calculating the total travel distance:

\[
\text{Total curve distance} = (n_s - 1) \times FF' \times \text{Total linear distance} = n_s \times EF
\]  \hspace{1cm} (A.1.12)

\[
\text{Total travel distance} = \text{Total curve distance} + \text{Total linear distance}
\]  \hspace{1cm} (A.1.13)

The above analysis (A.1.1–A.1.13) represented the most general solution of equipment turn path and it generated both the \(\Omega\)-turn for the harvester and the U-turn for the baler, based on the equipment turning radius and the gap between run paths.

### A.2. ABP turning path derivation

The overall methodology of equipment turning path while aggregating the bales, using the simplest case, is described (Fig.A.2). The other turning cases based on the location of the bales to be collected follow a similar strategy but with the corresponding variation. Two of the cases, as examples, are discussed in detail with the derivations and others in the form of tables in this section.

The path along the two bale locations B and C starting from A is represented as a first straight \(\overline{AD}\) path, then the second arc \(\overline{DG}\) path, and followed by the second straight \(\overline{GH}\) path. It can be visualized that if the second straight path \(\overline{GH}\) is rotated by \(\alpha_{GJ}\) counterclockwise, it will become vertical and resemble the first straight path \(\overline{AD}\) and evaluated as usual (Fig.A.2). Thus, the repeating path for any number of bales in a group will be the first straight and the second arc portions of the path (e.g., \(\overline{AD} + \overline{DG}\)).

Based on the location of C, which may fall anywhere around B (quadrant I through IV) gave rise to several unique cases of path derivation. The four major family of cases, derived based on the position of C were: ‘Up-Right’ (I quadrant, above and right of B, Fig.A.3), ‘Up-Left’ (II quadrant, above and left of B, Fig.A.4), ‘Down-Right’ (IV quadrant, below and right of B), and ‘Down-left’ (III quadrant, below and right of B). These major cases were further divided into 17 unique cases based on the positions of points of projections J (of G) and K (of H) with respect the center of arc F and projection I (of C) along the horizontal line through B (Supplementary material A.2–A.4).

If two bales lie too close, and fall within the turning path circle \((\overline{DF} = \overline{GF} = r_t)\) of the equipment, it will not be possible to collect the second bale without reversing the equipment. Thus, when \(\overline{FH} < (r_t + r_b)\) for right cases and \(\overline{FH} < (r_t - r_b)\) for left cases, the second bale was skipped. However, closer lying bales away from the turning path circle can be collected without
A.2. A simplest case of equipment turning to collect two bales (B & C); points A = starting location, B and C = bale points; \( r_t \) = turning radius; \( r_b \) = bale radius or picker arm length — same as Figure 4 and is reproduced for ready reference.

reversing the equipment. To address this issue, two nearest bales from a given location were always selected and processed for aggregation. The unpicked closer bale will be picked in the later part of the same trip or on another trip based on the bale coordinates. The derivations of ABP with two representative cases of right-up (I-quadrant; Fig.A.3) and left-down (III-quadrant; Fig.A.4) are presented hereunder.

A.2.1. Up-right I-quadrant turning case

Known parameters: Coordinate values of the points A, B, and C. Turning radius of the vehicle, \( r_t \), and the bale radius \( r_b \) (Fig.A.3).

To find: Points D, F, H, G, sweep angle, and start angle.

Finding point D\((x_D, y_D)\):

Calculating the distance of AB, and BC:

With respective to the point B,

\[ x_d = r_b; \quad y_d = 0 \]

With respective to the origin,

\[ x_D = x_2 + r_b; \quad y_D = y_2 \quad (A.2.1) \]

Finding point F\((x_F, y_F)\):

\[ AD = \sqrt{(x_1 - x_D)^2 + (y_1 - y_D)^2}; \quad BF = FN = r_t + r_b \]

With respective to the point B,

\[ x_f = BF; \quad y_f = 0 \]
Fig. A.3. Case: Up and Right, Turning case ABP using three known bale points, turning radius \( r_t \) and bale radius \( r_b \).

With respective to the origin,

\[
x_F = x_2 + x_f; \quad y_D = y_2 + y_f
\]  

(A.2.2)

Finding point H\((x_H, y_H)\):

\[
FC = \sqrt{(x_3 - x_F)^2 + (y_3 - y_F)^2}; \quad NC = GH = \sqrt{(FC)^2 - (r_t + r_b)^2}; \quad FH = \sqrt{NC^2 + r_t^2}
\]

\[
\alpha_{NC} = \tan^{-1}\left(\frac{FC}{NH}\right), \quad \alpha_{CI} = \cos^{-1}\left(\frac{x_3 - x_F}{FC}\right), \quad \alpha_{GH} = \tan^{-1}\left(\frac{GH}{r_t}\right)
\]

\[
\alpha_{HK} = \alpha_{CI} - (\alpha_{GH} - \alpha_{NC}); \quad FK = FH \times \cos(\alpha_{HK}); \quad HK = FH \times \sin(\alpha_{HK})
\]

With respective to the point B,

\[
x_h = r_t + r_b + FK; \quad y_h = HK
\]

With respective to the origin,

\[
x_H = x_2 + x_h; \quad y_H = y_2 + y_h
\]  

(A.2.3)

Finding sweep angle, \( \gamma_s \):

\[
\gamma_s = 180 - (\alpha_{CI} + \alpha_{NC})
\]  

(A.2.4)

Finding point G\((x_G, y_G)\):

\[
\alpha_{GJ} = \alpha_{CI} + \alpha_{NC}
\]

\[
FJ = r_t \times \cos(\alpha_{GJ}); \quad GJ = r_t \times \sin(180 - \gamma_s)
\]

With respective to the point B,

\[
x_g = FJ; \quad y_g = GJ
\]

With respective to the origin,

\[
x_G = x_2 + x_g; \quad y_G = y_2 + y_g
\]  

(A.2.5)

Start angle, \( \gamma_t \):
\[ \gamma_t = 180 - \gamma_s \quad (A.2.6) \]

Calculating the length of the curve, DG:
Using length of an arc formula,
\[ DG = \frac{\gamma_s}{360} \times 2\pi r_t \quad (A.2.7) \]

Calculating the linear distances, AD and GH:
\[ AD = \sqrt{(x_1 - x_D)^2 + (y_1 - y_D)^2} \quad (A.2.8) \]
\[ GH = \sqrt{(x_G - x_H)^2 + (y_G - y_H)^2} \quad (A.2.9) \]

Calculating the distance traveled:
In the event of collecting a bale located at up and right with respective to the points A and B, the distance traveled by the vehicle is:
\[ \text{Distance traveled} = AD + DG + GH \quad (A.2.10) \]

A.2.2. Down-left, III-quadrant turning case

Known parameters: Coordinate values of the points A, B, and C. Turning radius of the vehicle, \( r_t \), and the bale radius \( r_b \) (Fig. A.4).

Fig. A.4. Case: Down and Left, Turning case ABP using three known bale points, turning radius \( (r_t) \) and bale radius \( (r_b) \).

To find: Points D, F, H, G, sweep angle, and start angle.
Finding point \( D(x_D, y_D) \):
With respective to the point B,
\( x_d = r_b; y_d = 0 \)

With respective to the origin,

\[
x_D = x_2 + r_b; \quad y_D = y_2
\]  

(A.2.11)

Finding point \( F(x_F, y_F) \):

\[
AD = \sqrt{(x_1 - x_D)^2 + (y_1 - y_D)^2}; \quad BF = FN = r_t - r_b
\]

With respective to the point \( B \), \( x_f = BF; y_f = 0 \)

With respective to the origin,

\[
x_F = x_2 - x_f; \quad y_D = y_2 + y_f
\]  

(A.2.12)

Finding point \( H(x_H, y_H) \):

\[
FC = \sqrt{(x_3 - x_F)^2 + (y_3 - y_F)^2}; \quad NC = GH = \sqrt{(FC)^2 - (r_t - r_b)^2}; \quad FH = \sqrt{NC^2 + r_t^2}
\]

\[
\alpha_{NC} = \tan^{-1}\left(\frac{NC}{FN}\right); \quad \alpha_{CI} = \cos^{-1}\left(\frac{x_3 - x_F}{FC}\right); \quad \alpha_{GH} = \tan^{-1}\left(\frac{GH}{r_t}\right)
\]

\[
\alpha_{HK} = (\alpha_{CI} - \alpha_{NC}) + \alpha_{GH}; \quad FK = FH \times \cos(\alpha_{HK}); \quad HK = FH \times \sin(\alpha_{HK})
\]

With respective to the point \( B \),

\[
x_h = (r_t - r_b) + FK; \quad y_h = HK
\]

With respective to the origin,

\[
x_H = x_2 - x_h; \quad y_H = y_2 - y_h
\]  

(A.2.13)

Finding sweep angle, \( \gamma_s \):

\[
\gamma_s = 180 + (\alpha_{CI} - \alpha_{NC})
\]  

(A.2.14)

Finding point \( G(x_G, y_G) \):

\[
\alpha_{GJ} = \alpha_{CI} - \alpha_{NC}
\]

\[
FJ = r_t \times \cos(\alpha_{GJ}); \quad GJ = r_t \times \sin(180 - \gamma_s)
\]

With respective to the point \( B \),

\[
x_g = FJ; \quad y_g = GJ
\]

With respective to the origin,

\[
x_G = x_2 - x_g; \quad y_G = y_2 + y_g
\]  

(A.2.15)

Start angle, \( \gamma_t \):

\[
\gamma_t = 0
\]  

(A.2.16)

Calculating the length of the curve, \( DG \): Using length of an arc formula,

\[
DG = \frac{\gamma_s}{360} \times 2\pi r_t
\]  

(A.2.17)

Calculating the linear distances, \( AD \) and \( GH \):

\[
AD = \sqrt{(x_1 - x_D)^2 + (y_1 - y_D)^2}
\]  

(A.2.18)

\[
GH = \sqrt{(x_G - x_H)^2 + (y_G - y_H)^2}
\]  

(A.2.19)

Calculating the distance traveled:
In the event of collecting a bale located at down and left with respective to the points A and B, the distance traveled by the vehicle is:

\[
\text{Distance traveled} = AD + DG + GH
\]  \hspace{1cm} (A.2.20)

**A.3. Geometrical path drawings of all ABP turning cases**

Using the turning radius of the ABP vehicle and three known bale points, 17 different cases were observed while moving the point C over the points A and B. Four main cases, Up and right, Up and Left, Down and left, and Down and right emerged based on the position of point C with respective to points A and B. For instance, the point C up and right to the points A and B gave arise to the Up and Right case. Point F is the center of the turning circle, while points D, G, and H are the tangent points. The points K, J, and I are the perpendicular drop from the points H, G and C to the line drawn at F. The positions of the points K, J, and I with respective to the point F gave arise to different cases within the main cases. These major four cases and the sub-cases are illustrated in Figs. A.5–A.8.
Fig. A.5. A. RegRt - Regular right (point J on right with respective to point F), B. RegLt - Regular left, (point J on left with respective to point F), C. StRt - Straight right, point K lies on the right, with respective to point F, D. StLt - Straight left, point K lies on the left of point F; bale radius ($r_b$) = 1.5 m, and turning radius ($r_t$) = 10 m.
Up-Left turn cases

Fig. A.6. A. StRt - Straight right (point I, K, and J lie on right with respective to point F), B. StLt - Straight left, (point I lies on left, and point K and J lie on the right with respective to point F), C. RegLt - Regular left, point I and K lie on the left, and point J lies on the right with respective to point F, D. RegLt - Regular left, point I, K, and J lie on the left of point F; bale radius ($r_b$) = 1.5 m, and turning radius ($r_t$) = 10 m.
Down-Left turn cases

Fig. A.7. RegUp - Regular up (point H, I, J and K lie above point F), B. RegDn - Regular down (point H lies below, and point I, J, and K lie below with respective to point C), C. RegMd - Regular middle (point H lies below and points I, J, and K lie on the same level and on the left side of point F), D. StLt - Straight left (points J and K lie on the left, and point I lies on the right with respective to point F), E. StRt - Straight right (points K and I lie on the right, and point J lies on the left with respective to point F); bale radius \( r_b = 1.5 \, \text{m} \), and turning radius \( r_t = 10 \, \text{m} \).
Fig. A.8. A. StLt - Straight left (points K and I lie on the left and point J lies on the right with respective to point F), B. StRt - Straight right (point K lies on the left, and points I and J lie on the right with respective to point F), C. RegDn - Regular down (points K, I, and J lie on the right of point F, and point G lies below point F), D. RegUp - Regular up (points K, I, and J lie on the right of point F, and point G lies above point F).
A.4. All ABP turning cases derived expressions and track impacted area simulation results

The consolidated derived expressions for all the 17 turning cases of ABP are presented in Tables A.1–A.2. Table A.3 shows the simulation results of impacted area obtained across different areas (8–259) and bales/trip (1–23).
<table>
<thead>
<tr>
<th>Case</th>
<th>Item</th>
<th>RegUp $C^+H^+_U G^+_U$</th>
<th>RegDn $C^+H^+_U G^-_U$</th>
<th>RegMd $C^+H^+_D G^-_D$</th>
<th>StLt $C^+H^+_L G^-_L$</th>
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<td>$x_H = (r_t - r_b) - FK$</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>G</td>
<td>$\alpha_{GJ} = 180 - (a_{CI} + a_{NC})$</td>
<td>$\alpha_{GJ} = 180 - (a_{CI} + a_{NC})$</td>
<td>$\alpha_{GJ} = 180 - (a_{CI} + a_{NC})$</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$\gamma_s = 180 - (a_{CI} + a_{NC})$</td>
<td>$\gamma_s = 180 - (a_{CI} + a_{NC})$</td>
<td>$\gamma_s = 180 - (a_{CI} + a_{NC})$</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$\gamma_t = 0$</td>
<td>$\gamma_t = 0$</td>
<td>$\gamma_t = 0$</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

RegUp - Regular up, RegDn - Regular down, RegMd - Regular middle, RegRt - Regular right, RegLt - Regular left, StLt - Straight Left, StRt - Straight Right; $C^+$ - point C lies on the right with respective to point F, $C^-$ - point C lies on the left with respective to point F, $H^+$ - point H lies on the right with respective to point F, $H^-$ - point H lies on the left with respective to point F, $G_U$
Table A.2. Conditions of ABP turning cases used in the simulation for turning parameters derivation.

<table>
<thead>
<tr>
<th>Case</th>
<th>CaseName</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>DnLt</td>
<td>RegUp C⁻H⁻G⁻</td>
<td>((x_C \leq x_F) &amp; (((\alpha_{NC} - \alpha_{CL}) \leq \alpha_{GH}))</td>
</tr>
<tr>
<td></td>
<td>RegDn C⁺H⁺G⁺</td>
<td>((x_C \leq x_F) &amp; (((\alpha_{NC} - \alpha_{CL}) \leq \alpha_{GH})) &amp; (\alpha_{NC} \geq \alpha_{CL}))</td>
</tr>
<tr>
<td></td>
<td>RegMd C⁻H⁻G⁺</td>
<td>((x_C \leq x_F) &amp; (((\alpha_{CL} - \alpha_{NC}) \leq \alpha_{GH})) &amp; (\alpha_{NC} &lt; \alpha_{CL}))</td>
</tr>
<tr>
<td></td>
<td>StLt C⁺H⁺G⁻</td>
<td>((x_C &gt; x_F) &amp; (((\alpha_{CL} - \alpha_{NC}) \leq \alpha_{GH})) &amp; (\alpha_{NC} &lt; \alpha_{CL})) &amp; (((\alpha_{CL} + \alpha_{NC}) - \alpha_{GH}) &gt; 90))</td>
</tr>
<tr>
<td></td>
<td>StRt C⁺H⁺G⁺</td>
<td>((x_C &gt; x_F) &amp; (((\alpha_{CL} - \alpha_{NC}) &lt; \alpha_{GH})) &amp; (\alpha_{NC} &lt; \alpha_{CL})) &amp; (((\alpha_{CL} + \alpha_{NC}) - \alpha_{GH}) &lt; 90))</td>
</tr>
<tr>
<td>DnRt</td>
<td>RegUp C⁺H⁺G⁺</td>
<td>((x_C \geq x_F) &amp; (\alpha_{NC} \geq \alpha_{CL}))</td>
</tr>
<tr>
<td></td>
<td>RegDn C⁺H⁺G⁺</td>
<td>((x_C \geq x_F) &amp; (\alpha_{NC} &lt; \alpha_{CL})) &amp; (((\alpha_{CL} - \alpha_{NC} + \alpha_{GH}) \leq 90)))</td>
</tr>
<tr>
<td></td>
<td>StLt C⁺H⁻G⁺</td>
<td>((x_C \geq x_F) &amp; (\alpha_{NC} &lt; \alpha_{CL})) &amp; (((\alpha_{CL} - \alpha_{NC} + \alpha_{GH}) &gt; 90)))</td>
</tr>
<tr>
<td></td>
<td>StRt C⁻H⁻G⁺</td>
<td>((x_C &lt; x_F) &amp; (\alpha_{NC} &lt; \alpha_{CL})) &amp; (((180 - (\alpha_{CL} + \alpha_{NC})) + \alpha_{GH}) &gt; 90))</td>
</tr>
<tr>
<td>UpRt</td>
<td>StLt C⁻H⁻</td>
<td>((x_C \leq x_F) &amp; (((\alpha_{CL} - \alpha_{NC} + \alpha_{GH}) \leq 90)))</td>
</tr>
<tr>
<td></td>
<td>StRt C⁺H⁺</td>
<td>((x_C \leq x_F) &amp; (((\alpha_{CL} - \alpha_{NC} + \alpha_{GH}) &gt; 90))) &amp; ((y_C \geq F_y + r_{tb})))</td>
</tr>
<tr>
<td></td>
<td>RegRt C⁺H⁺RC</td>
<td>((x_C &gt; x_F) &amp; (y_C \geq F_y + r_{tb}))</td>
</tr>
<tr>
<td></td>
<td>RegLt C⁺H⁺LC</td>
<td>((x_C &lt; x_F) &amp; (y_C &lt; F_y + r_{tb}))</td>
</tr>
<tr>
<td>UpLt</td>
<td>RegRt C⁻H⁻RC</td>
<td>((x_C \leq x_F) &amp; (((180 - (\alpha_{CL} + (\alpha_{NC} - \alpha_{GH})) \geq 90))) &amp; ((y_C \geq F_y + r_{tb})))</td>
</tr>
<tr>
<td></td>
<td>RegLt C⁻H⁻LC</td>
<td>((x_C \leq x_F) &amp; (((180 - (\alpha_{CL} + (\alpha_{NC} - \alpha_{GH})) \geq 90))) &amp; ((y_C &lt; F_y + r_{tb})))</td>
</tr>
<tr>
<td></td>
<td>StLt C⁻H⁺</td>
<td>((x_C \leq x_F) &amp; (((180 - (\alpha_{CL} + (\alpha_{NC} - \alpha_{GH})) &lt; 90)))</td>
</tr>
<tr>
<td></td>
<td>StRt C⁺H⁺</td>
<td>((x_C &gt; x_F) &amp; (((\alpha_{CL} - \alpha_{NC}) + \alpha_{GH}) &lt; 90)))</td>
</tr>
</tbody>
</table>

RegUp - Regular up, RegDn - Regular down, RegMd - Regular middle, RegRt - Regular right, RegLt - Regular left, StLt - Straight Left, StRt - Straight Right; C⁻ - point C lies on the right with respective to point F, C⁺ - point C lies on the left with respective to point F, H⁻ - point H lies on the right with respective to point F, H⁺ - point H lies on the left with respective to point F, G⁻, G⁺.
Table A.3. Simulation results of aggregation equipment track impacted area obtained for different field areas and bales/trip.

<table>
<thead>
<tr>
<th>Area (ha)</th>
<th>Number of Bales</th>
<th>Whole Tractor 1</th>
<th>Fractional Tractor 2</th>
<th>Total Tractor 1</th>
<th>ABP Tractor 1</th>
<th>ABP Tractor 2</th>
<th>Curve Increase Impacted area</th>
<th>ABP difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>101</td>
<td>2.35</td>
<td>41.75</td>
<td>22.7</td>
<td>47.7</td>
<td>33.40</td>
<td>18.16</td>
<td>33.40</td>
</tr>
<tr>
<td>16</td>
<td>187</td>
<td>3.29</td>
<td>40.04</td>
<td>22.16</td>
<td>42.12</td>
<td>39.28</td>
<td>19.56</td>
<td>39.28</td>
</tr>
<tr>
<td>24</td>
<td>275</td>
<td>4.15</td>
<td>48.72</td>
<td>25.32</td>
<td>45.16</td>
<td>42.12</td>
<td>21.88</td>
<td>42.12</td>
</tr>
<tr>
<td>32</td>
<td>375</td>
<td>5.01</td>
<td>57.40</td>
<td>28.72</td>
<td>53.52</td>
<td>50.48</td>
<td>24.16</td>
<td>50.48</td>
</tr>
<tr>
<td>40</td>
<td>475</td>
<td>5.87</td>
<td>66.08</td>
<td>32.12</td>
<td>61.91</td>
<td>59.13</td>
<td>26.44</td>
<td>59.13</td>
</tr>
<tr>
<td>65</td>
<td>710</td>
<td>7.10</td>
<td>84.80</td>
<td>42.40</td>
<td>80.80</td>
<td>78.00</td>
<td>30.64</td>
<td>78.00</td>
</tr>
<tr>
<td>85</td>
<td>950</td>
<td>8.35</td>
<td>103.56</td>
<td>51.76</td>
<td>99.80</td>
<td>96.91</td>
<td>36.84</td>
<td>96.91</td>
</tr>
</tbody>
</table>

Note: The table continues with similar data for other areas, trips, and bale counts.
A.5. ABP impacted area prediction results using non-linear models

Plots of the results of the developed impacted area prediction models with field area (8–259 ha) and 11 levels of biomass yield (3–40 Mg/ha) for bales/trip 1 (tractor) and 23 (ABP) are presented (Fig.A.9). The prediction models exhibit a non-linear trend with good fit performance ($R^2 > 0.99$).
Fig. A.9. Fitted ABP impacted area non-linear models with area ranging from 8–259 ha and biomass yield levels ranging from 3–40 Mg/ha. A. ABP impacted area model, bales/trip = 1 (minimum); B. ABP impacted area model, bales/trip = 23 (maximum). Bales/trip = 8 (optimum) is presented already in Fig.15 of the article.